Low Temperature Diamagnetism of Electrons in a Cylinder, II

William Band The State College of Washington, Pullman, Washington

In a paper under the above title (1) the wave functions for an electron in a magnetic field and cylindrical enclosure were expressed as modified Bessel functions whose zeroes could be calculated as functions of the magnetic field. It has been pointed out (2) that terms to second order in the field should have been retained in the expression for the wave functions to give a consistent theory of the susceptibility. When this is done, Eq.(14), Ref.1 becomes

$$J^*_m(x) = \left[1 + \left\{\tfrac{1}{2}\beta + \tfrac{1}{3}(m+1)\beta^2\right\}x^2 + \tfrac{1}{4}\beta^2x^4\right] J_m(x) - \tfrac{1}{3}\beta^2x^2J_{m-1}(x) + \tfrac{2}{3}\beta^2(m^2-1)xJ_{m+1}(x) \qquad (1)$$

where β is defined in Ref.1, and is proportional to the field. The zeroes of this function are easily evaluated to terms of order β^2, and we find the first zero to be at

$$x^*_{m1} = x_{m1} + a^4\alpha^2/m \qquad (2)$$

where x_{m1} is the unmodified Bessel function zero, a is the radius of the cylinder, and $\alpha = eH/2\hbar c$. This modification could lead to only minor numerical changes in the results of Ref.1. For large values of j we find

$$x^*_{mj} = x_{mj} + \tfrac{1}{3}\gamma\, a^4\alpha^2\left[1 - 2(m^2-1)/x_{mj}{}^2\right] \qquad (3)$$

where

$$\gamma = \frac{1}{m}\left[1 - \frac{1}{1+m/2j}\right] \qquad (4)$$

Here again one might suppose that the modification becomes negligible at large m abd j values, but taking the square of x^*_{mj}, noting the asymptotic value of x_{mj}, and using the result in the energy expression eq.(12), Ref.1, we find to terms in α^2:

$$2ME_{kmj}/\hbar^2 = (\pi j/a)^2 + (\pi n/L)^2 + 2m\alpha + \tfrac{2}{3}\pi\gamma\, j a^2\alpha^2\left[1 - 2(m^2-1)/x_{mj}{}^2\right] \qquad (5)$$

The diamagnetic term in α^2 is not a constant on the Fermi surface, it has a maximum value when $j \gg m$, $\gamma \to 1/2j$; and a minimum value where m has its maximum value, $m \sim \pi j$, $\gamma \to 1/(2+\pi)j$. This energy spectrum is now similar with that obtained by Dingle (3) using a perturbation approach to the wave functions, except that his diamagnetic term was constant on the Fermi surface, and is the same as in (5) with $\gamma = 1/2\pi j_\rho$

While therefore there remain minor numerical differences, the more violent conflict between Ref.1 and Dingle's work is apparently resolved, and the integration approximation method of studying the susceptibility used in Ref. 1 is of no further interest: a more exact discussion of the susceptibility seems called for in terms of the spectrum (5) and wavefunctions obtained from (1).

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1). Physical Review 91, 249, 1953

2). Private communication from Dr. F.S.Ham, Harvard University

3). Proc. Roy.Soc.London A212, 47, 1953, Eq.(1.8)

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